PRACTICAL CONSIDERATIONS ABOUT ERROR ANALYSIS FOR DISCRETE EVENT SIMULATIONS MODEL

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Contents

- Introduction about discrete event simulation models;
- Purpose of the work and methodology;
- EOQ model as a “test simulation model”;
- Design of the experiments;
- Findings;
- Conclusions;

Discrete Event Simulation

In discrete-event simulation (DES), the operation of a system is represented as a chronological sequence of events[1]. Each event occurs at an instant in time and marks a change of state in the system.

In addition to the representation of system state variables and the logic of what happens when system events occur, discrete event simulations include the following components:

- **Clock**: the simulation must keep track of the current simulation time, in whatever measurement units are suitable for the system being modeled;
- **Events List**: the simulation maintains at least one list of simulation events. This is sometimes called the “pending event set” because it lists events that are pending as a result of previously simulated event;
- **Random-Number Generators**: the simulation needs to generate random variables of various kinds, depending on the system model;
- **Statistics**: the simulation typically keeps track of the system's statistics, which quantify the aspects of interest, KPI (Key Performances Indicators);
- **Ending Condition**: theoretically a discrete-event simulation could run forever. So the simulation designer must decide when the simulation will end.

[1]*Stewart Robinson (2004). Simulation - The practice of model development and use. Wiley.*

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Discrete Event Simulation

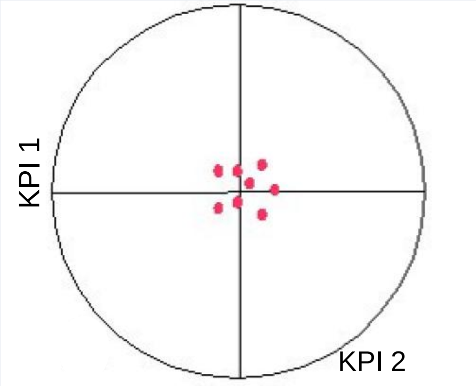
A Discrete Event Simulation model is a computer code developed in a specific language (C++, MathLab, SciLab, AutoMod, ecc).

Main loop of a discrete event simulation code

Start
- Initialize **system state variables** that defined the **simulated scenario**.
- Initialize Clock (usually starts at simulation time zero).

"Do loop"
- While (Ending Condition is FALSE) then do the following:
- Set clock to next event time.
- Update statistics.

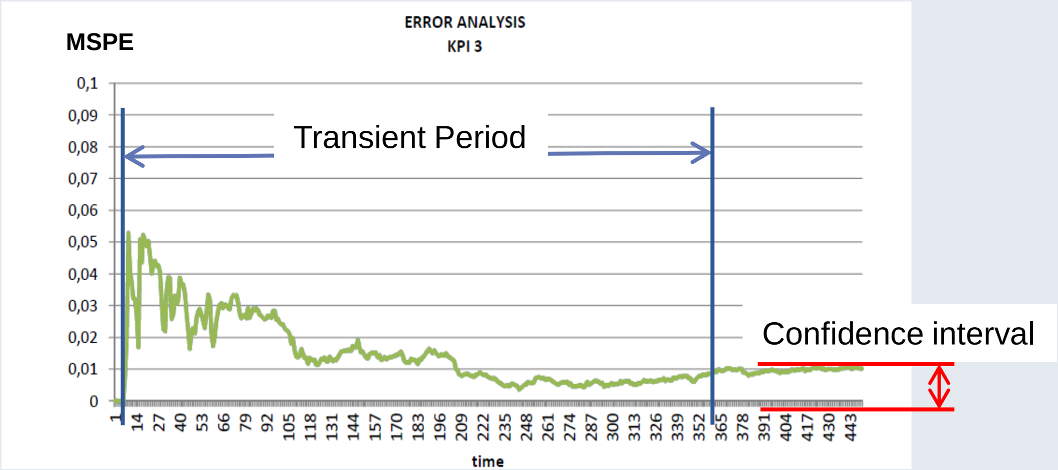
End
- Generate statistical report of considered KPI.



Because of Random Numbers used in the main "Do loop" the results in term of KPI varies from a simulation to another even if the system state variables are initialized at the same values. This variance depends on both: system itself and simulation model architecture.

Simulation outputs reliability

Classic error analysis consists of individuating the initial transient period and the confidence interval for each KPI, the MSPE (Mean Square Pure Error) method is used.



$$MSPE_i = \frac{\sum_{j=1}^{r}(y_{ji} - Y_i)^2}{r-1}$$

- i is the simulated day;
- y_{ji} is the output y value at day i and for replication j;
- Y_i is the mean of the output y at day i on the r replications;

… but how it behaves when the simulated scenario is changed?

The variance of outputs confidence interval between different scenarios is often faced with the hypothesis that it is normally distributed around a central value used in the reference scenario.
But in many practical cases there is no evidences that this hypothesis is correct and, moreover, the significance of central value, for the reference scenario, is lost.

Purpose of the paper

<u>The paper is focused on the study of confidence interval variance related to the variance of simulated scenario.</u> The aim of this paper is to give some practical guidelines in order to drive the error analysis for discrete event stochastic simulation models.

Methodology: a rather simple system is considered to develop a discrete event simulation model and the MSPE method is used to estimate outputs confidence interval. Then simulations are performed according to different scenarios and the variance of confidence interval is studied for different KPI:

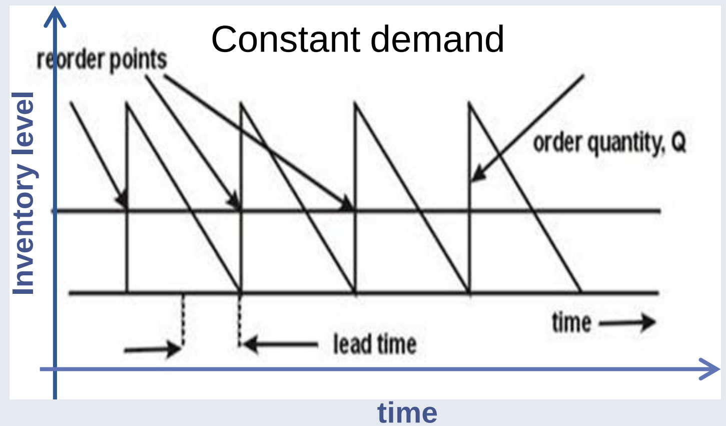
1. <u>A discrete event simulation model is developed according with the standard EOQ model</u> (single item with normal distributed demand) taken as reference scenario ;
2. <u>A set of different KPI is defined</u>;
3. <u>Simulations are performed according to different scenarios</u>;
4. <u>Variance of confidence interval is studied for different scenarios</u> related to each KPI;

Economic Order Quantity model

The economic order quantity (EOQ) is a well-known and commonly used re-order point inventory control techniques.

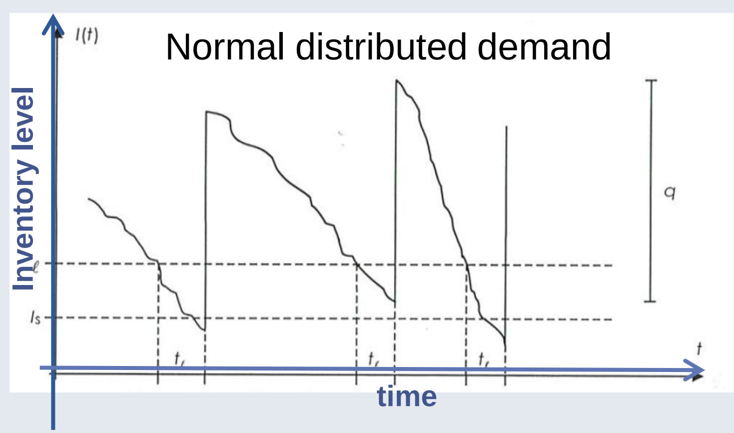
The model variables:

- D: demand, pieces per unit time (year);
- C: order cost per pieces;
- h: holding costs per pieces per unit time;
- L_t: lead time in unit time (days);
- Q: order quantity;
- SS: safety stock to achieve the target service level.



The main model hypothesis:

- <u>the demand D is constant or normal distributed</u>;
- the ordered quantity arrives all at once and it is instantly available;
- no shortage are allowed;
- costs are time and quantity invariant;
- <u>lead time is fixed or normal distributed</u>;
- there are no quantity constrains;



Simulation model set

The simulation model is developed according with EOQ model and variables are set to define the reference scenario expected from the theoretical model.

Different KPI are defined in order to evaluate system performances in term of service level.

Symbol and definitions

Symbol	Unit	Definition
N	Day	Number of days for simulation
D_i	Unit/day	Mean demand per day in units
Lt	Day	Mean lead time in day
C_o	Euro/order	Single order cost in euro
C_s	Euro/ unit*year	Stock cost in euro per unit per year
SS	Unit	Safety stocks in unit

Used parameters set

Parameter	Set value
D_i	1.000,00
σ_d	300,00
Lt	7,00
σ_t	2,00
C_o	1.000,00
C_s	1,00
Imposed SL	0,95

Used KPI

KPI	Unit	Definition
SL1	%	1-Number of stock-out in days per day
SL2	%	1-Number of stock-out in units per day
SL3	%	1-Number of stock-out in units per day during lead time
SL4	%	1-Number of stock-out event during lead time period

4 different definitions of "Service Level" are used as KPI

SL4 is the definition used in the theoretical model

Design of the experiments

To investigate the influence of different parameters on confidence intervals four factors are considered. These four factors are:

- Demand distribution;
- Lead time distribution;
- Ratio C_o/C_s (order cost/stock cost);
- SS, safety stocks.

Theoretical EOQ model, simulation model results fit with theory

Factors setting

	Low (-1)	Mean (0)	High (+1)
D	Normal distribution, mean = 1,000 units/day, standard deviation = 300 units/day	Uniform distribution, minimum = 500 units/day, maximum = 1,500 units/day	Exponential distribution, mean = 1,000 units/day
Lt	Normal distribution, mean = 7 day, standard deviation = 2 day	Uniform distribution, minimum = 1day, maximum = 13 day	Exponential distribution, mean = 7 day
C_o/C_s	100	1.000	1.900
SS	0 units	1.000 units	2.000 units

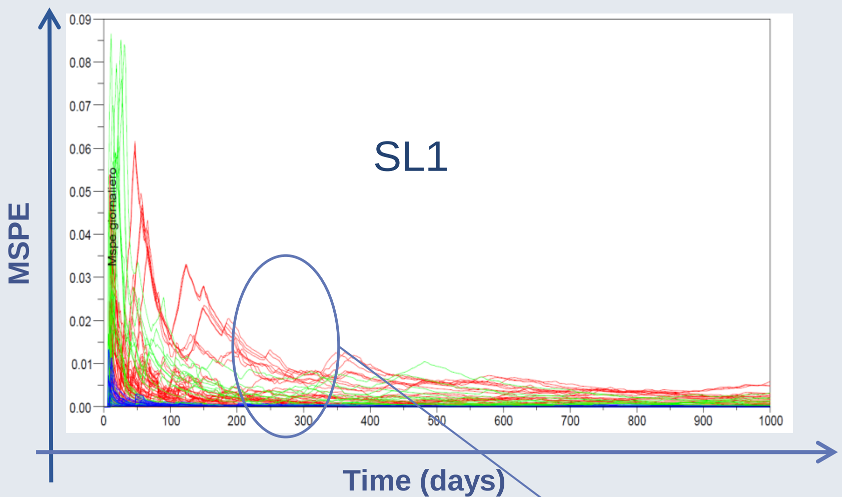
Non-standard EOQ model, simulation model results are useful to predict model behavior

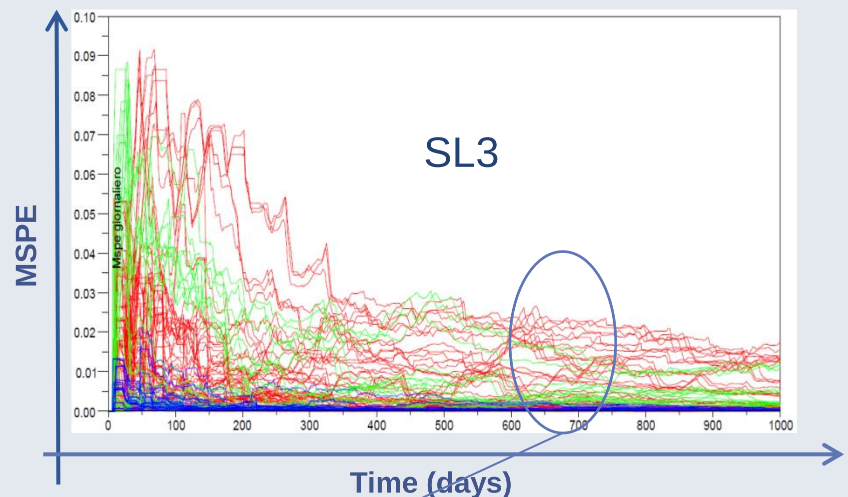
A full factorial experiment with three levels is used in this paper. Four factors and three levels give $3^4 = 81$ combinations. For each combination a number of 5 replications were conducted for a number of 405 simulations.

Findings (stability of the results)

Experiments are evaluated in terms of stability of the results and confidence interval width for all considered KPI.

The simulations are conducted for a length of 1.000 days, the initial transient period length varies according with different parameters set and the variance is more significant for certain KPI.





81 graphs from the full factorial combinations experiments

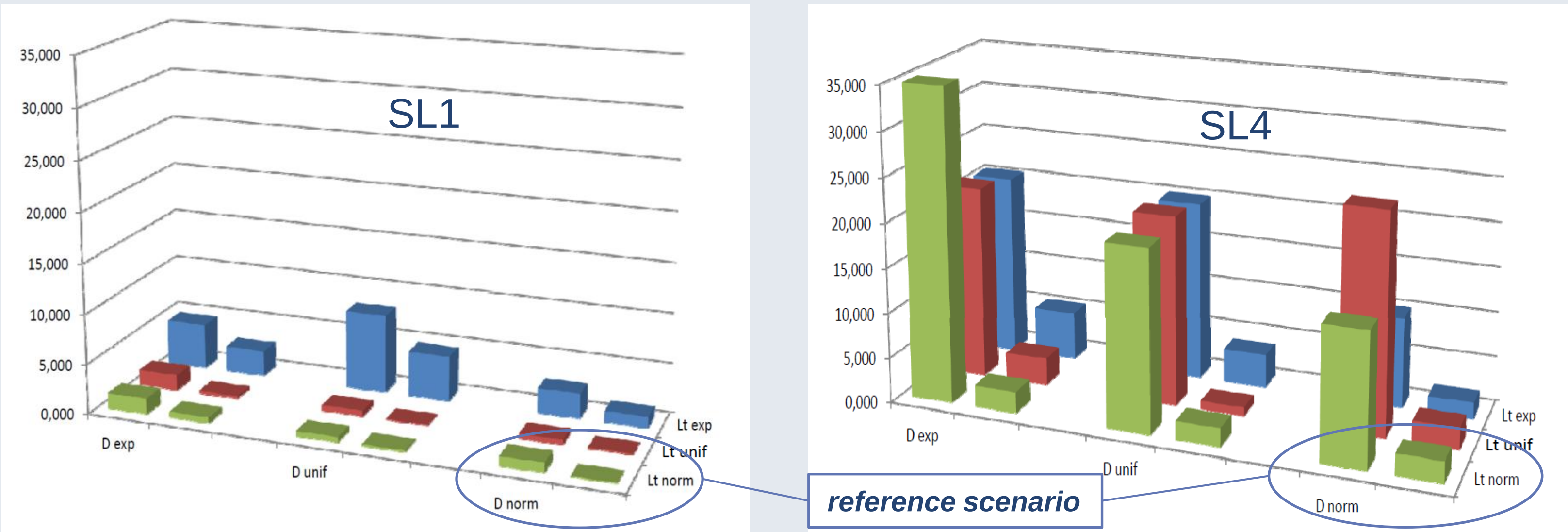
Findings (confidence interval width)

To evaluate the significance of confidence intervals the results are presented for each KPI as the ratio between half interval and the mean for each KPI. Confidence half intervals are calculated for a 95% level of significance.

Factors	LS1	LS2	LS3	LS4
F_tipoD	3,15E-14 ***	4,99E-14 ***	4,85E-15 ***	0,0002638 ***
F_tipoLt	< 2.2e-16 ***	< 2.2e-16 ***	< 2.2e-16 ***	0,3792613
F_SS	0,004364 **	0,013121 *	0,0001933 ***	8,62E-09 ***
F_CC	0,132156	0,126539	< 2.2e-16 ***	3,49E-07 ***
F_tipoD:F_tipoLt	< 2.2e-16 ***	< 2.2e-16 ***	4,03E-16 ***	0,1436708
F_tipoD:F_SS	0,639589	0,952472	0,8427638	0,9926262
F_tipoD:F_CC	0,016213 *	0,031195 *	0,0091386 **	0,0912483
F_tipoLt:F_SS	0,570304	0,801933	0,0424879 *	0,2119648
F_tipoLt:F_CC	0,025063 *	0,009775 **	< 2.2e-16 ***	0,1682513
F_SS:F_CC	0,92423	0,918936	0,9432251	0,8817591

The ANOVA test reveals that the considered factors have different impact on confidence interval width. Demand and lead time distribution have a strong effects, , but this effect depends on the considered KPI.

Conclusions



The initial transient period and the related confidence interval depend in a very different way by the considered parameters:

- for numeric parameters, the hypothesis that confidence interval variance is normal distributed around a central value calculated in the reference scenario is almost verified;
- if not numeric parameters are involved, in this example distribution types, the confidence interval must be re-calculated in each scenario because the variance could be high and the interactions are almost unpredictable;
- different KPI can have a very different behavior, even if they are all related to a common concept.

Thanks for your attention !!!

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