A PROPOSED CORRECTION FOR THE EXPECTED SERVICE LEVEL IN THE ECONOMIC ORDER QUANTITY (EOQ) MODEL UNDER RELAXED DEMAND ASSUMPTIONS

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- Purpose of the work and methodology;
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The Economic Order Quantity (EOQ) model

A basic problem for manufacturers is, when ordering supplies, to determine what quantity of a given item to order and to determinate what quantity hold in stock to prevent stock out.

The economic order quantity (EOQ), first introduced by Harris, and developed by Brown and Bather with stochastic demand, is a well-known and commonly used inventory control techniques.

The standard EOQ results are easy to apply but are based on a number of unrealistic assumptions. One of this assumption is that the demand is normally distributed in any interval.

The EOQ model with steady demand

The main model hypothesis:

- the demand D is constant;
- the ordered quantity arrives all at once and it is instantly available;
- no shortage are allowed;
- costs are time and quantity invariant;
- lead time is fixed;
- there are no quantity constrains;

The variables:

- D: demand, pieces per unit time (year);
- C: order cost per pieces;
- h: holding costs per pieces per unit time;
- L_t: lead time in unit time (days);
- Q: order quantity. This is the variable we want to optimize;

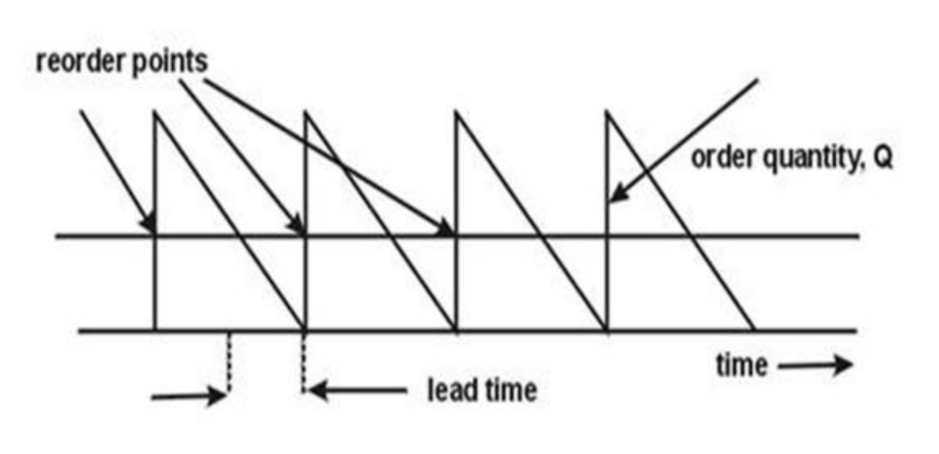
where the optimal solution is:

$$Q = \sqrt{\frac{2CD}{h}}$$

and the reorder point is:

$$B = \frac{D\ Lt}{n*}$$

* n is the number of days in a year in this case



The EOQ model with normally distributed demand

If the demand is not perfectly steady there are risks of shortages, to control this risk service level concept and safety stocks are introduced.

Safety Stocks (SS) are extra stock that are maintained to mitigate risk of stock outs due to uncertainties in supply and demand.

Service Level (SL) is the desired probability that a stock out does not occurred.

In the EOQ model with:

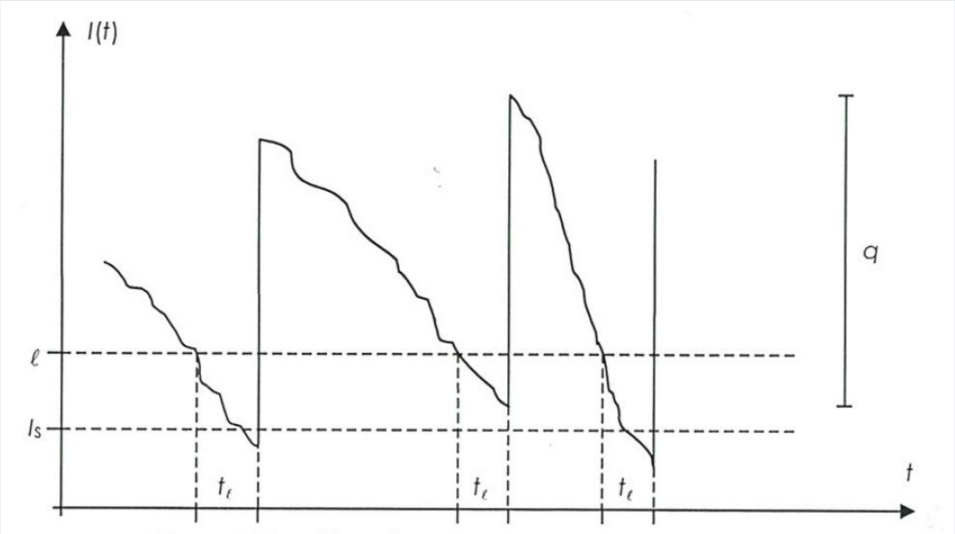
- normal distributed demand D in any interval of time;
- fixed lead time L_t.

The expression for safety stocks is:

$$SS = Z_\alpha \sqrt{\sigma_d^2 \, Lt}$$

where

$$Z_\alpha{}^{*} \propto SL$$



* Z_α is the cumulative probability of the standardized normal random variable, for SL = 95% the related Z_α = 1,645

Purpose of the paper

In several practical cases the assumption about independence of successive demands, and consequently demand normal distribution in any interval, is not supported by real data.

- The main purpose is to investigate the effects on the expected service level (SL) after relaxing normal distribution assumption on demand.

Moreover, in many practical cases the used time sample to analyze the historical demand is not related to the lead time.

- The second purpose is to consider demand sampling problems in order to quantify the difference, in terms of Safety Stocks, related to the use of different time samples.

Methodology

1. A discrete event simulation model is developed according with the standard EOQ model (single item with normal distributed demand);
2. The model is validated on achieved service level;
3. The model is adapted to demand relaxed assumptions;
4. The results achieved are compared with the expected service level

The EOQ model with relaxed demand assumption

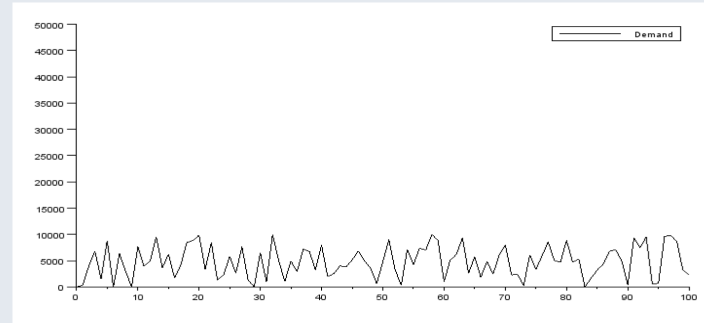
Relaxed demand assumptions:

1. The demand D_i is randomly generated for the i-period, according with the assumption of normal distribution period by period (the considered i-period is equal to a month);
2. The D_i period demand is split into a defined number (N_p) of single daily demand (d_k) according with the following assumption:
   - d_k must be higher than Min_d;
   - d_k must be lower than Max_d;
   - the d_k daily demand are uniformly distributed between Min_d and Max_d;
   - the total demand in the period is equal to D_i.

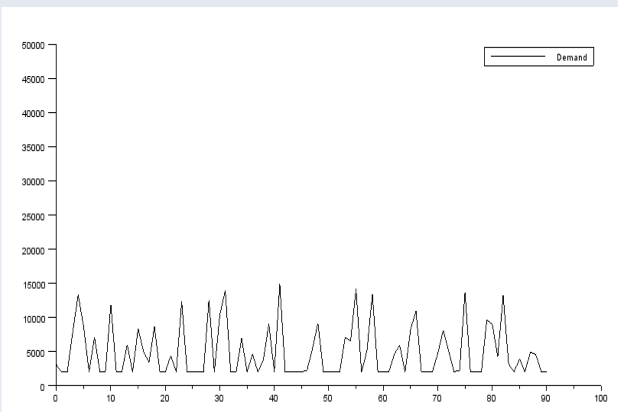
<u>The demand generated relaxes the classic assumption</u>, in fact the demand is normally distributed if the considered interval is long enough to include a large number of periods and the used time sample is equal to the period itself. If the considered interval is shorter or the time sample is different <u>the presented assumptions don't assure demand normality in any interval of time.</u>

Generated demands

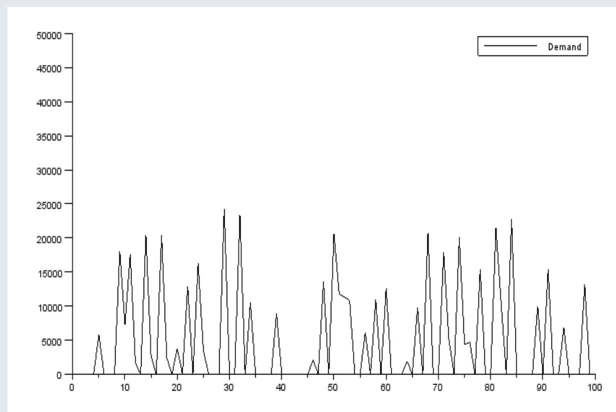
The demand charts show how varying Min_d and Max_d implies to study the EOQ model behavior at different degree of demand lumpiness.



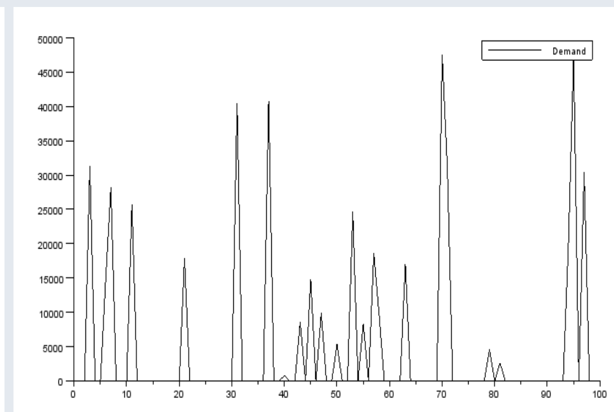
Generated demand with normal distribution hypothesis.



Generated demand with relaxed hypothesis, Min 2%, Max 15% of D_i



Generated demand with relaxed hypothesis, Min 0%, Max 25% of D_i



Generated demand with relaxed hypothesis, Min 0%, Max 50% of D_i

Demand sampling problems

The formula for safety stocks calculation is:

SS = $Z_\alpha \sqrt{\sigma_d^2 \, Lt}$

and, according with the EOQ model theory, σ_d is the standard deviation of the demand calculated using a time sample equal to the Lead time L_t.

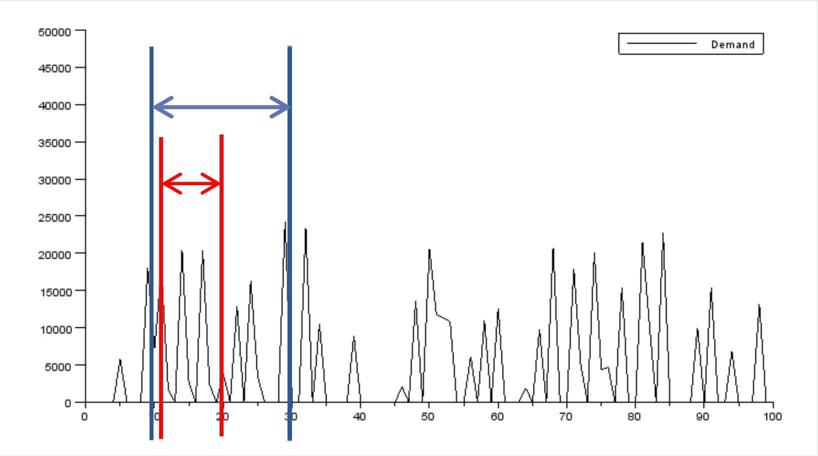
What happened, under relaxed assumptions, if a different time sample is used?

Two different time samples were used:

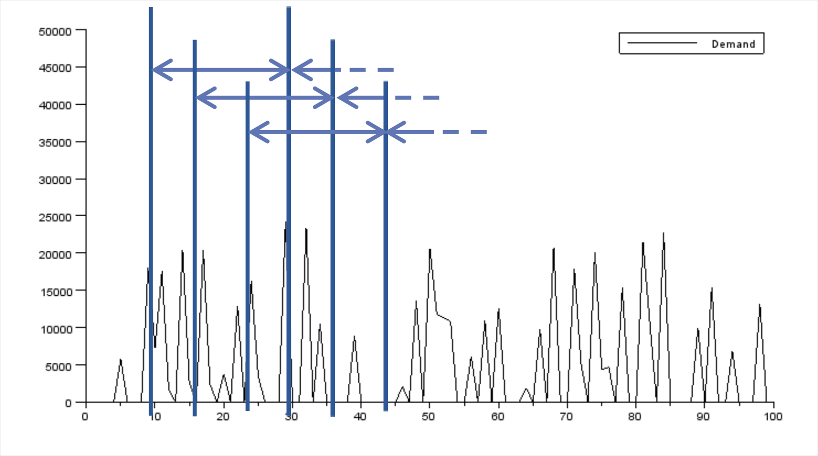
- the lead time, according with the standard theory;
- a period equal to a month, according with common incorrect industrial practice.

… and about the influence of the initial point, used for σ_d calculation, since under relaxed assumptions, the demand is not normal distributed in any interval?

Standard deviation σ_d was calculated N_p time and at each time the initial day is translated by one. The safety stocks were calculated according with the higher standard deviation σ_d.



Different time sample



Different initial point for the sample time sample

Design of the experiments

The simulation model parameters are set in order to be representative for small-medium manufactures operating in the food sector. The symbol definition and the used set is illustrated in the tables.

Symbol	Definition
D_i	Mean demand per i-period in unit
σ_i	Standard deviation for demand per i-period in unit
Np	Number of days for period
C_l	Set-up cost in euro per unit
C_s	Stock cost in euro per unit per year
Max_d	Maximum demand for a day in unit
Min_d	Minimum demand for a day in unit
Lt	Lead time in day
σ_t	Standard deviation for lead time in day

Parameter	Set value
D_i	100.000,00
Np	20,00
C_l	200,00
C_s	1,00
Lt	5,00
σ_t	0,00
Imposed Service Level	0,95

To investigate the influence of Min_d and Max_d (minimum and maximum demand admitted in a single day) at different level of σ_i (demand monthly standard deviation) all the other parameters of the model are set to specific values. The investigated ranges, for the three variables, are:

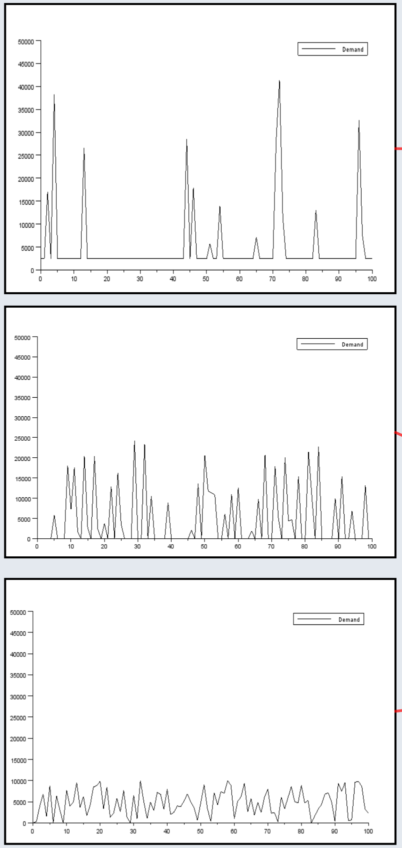
- Min_d between 0 to $D_i/2*N_p$ (in this case from 0 to 2.500 units, with a step of 500 units);
- Max_d from D_i/N_p to D_i (in this case from 5.000 to 100.000 units, with a step of 2.500 units);
- Demand monthly standard deviation σ_i varies from 0 up to the 20% of D_i (in this case from 0 to 20.000 units, with a step of 5.000 units).

Findings

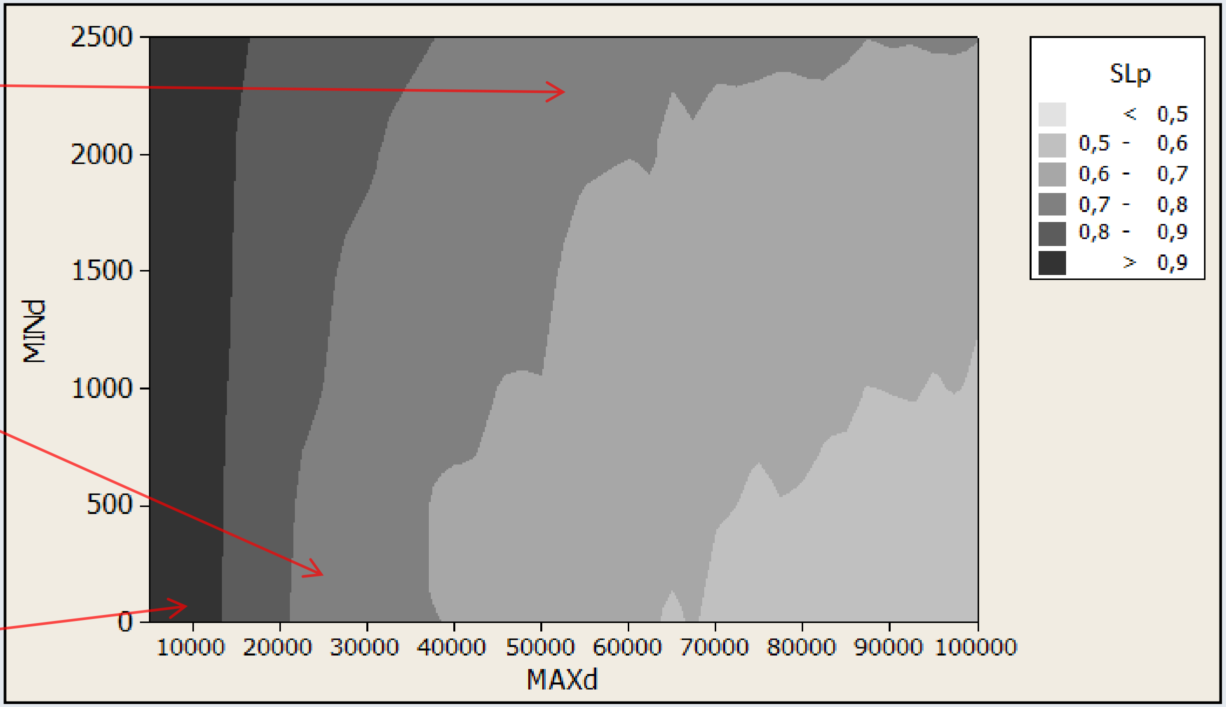
Results are shown in term of achieved service level, where the imposed SL was 95%:

- SL_p, as the rate between available units against total demanded units.

The classic definition of SL (the rate between the number of days when the out of stock occurred against total days), used in the EOQ model, is not representative under relaxed assumptions. In fact, if the normal distribution of the demand is not guarantee in any interval also the linkage between demanded quantity and days of availability is not guarantee.



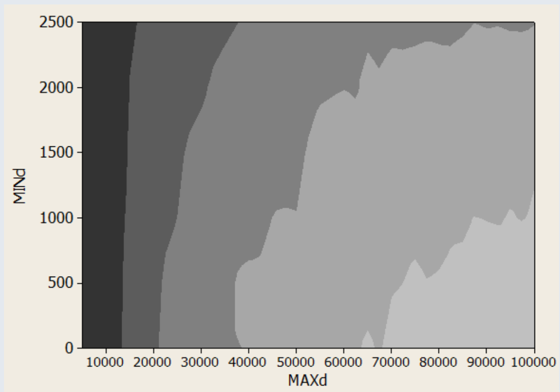
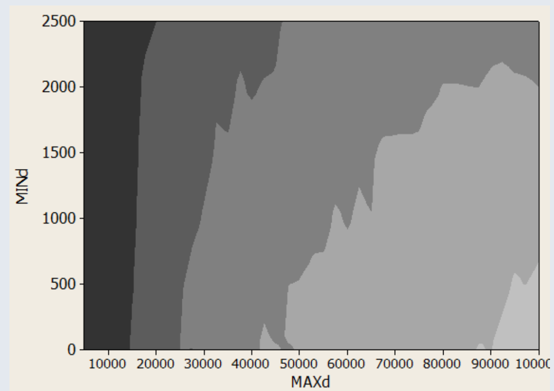
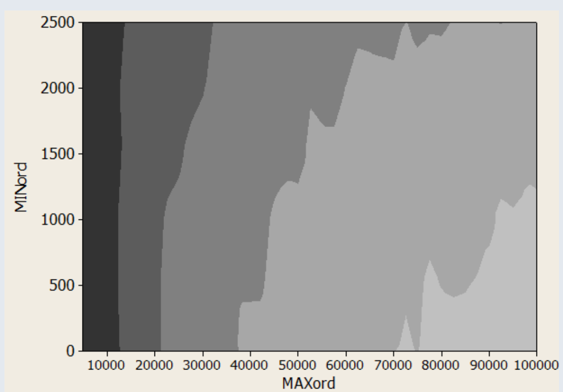
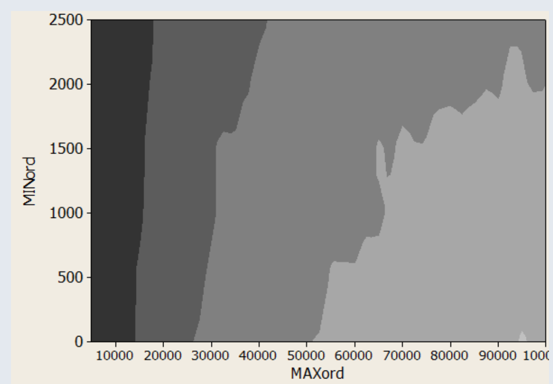
Generated demand

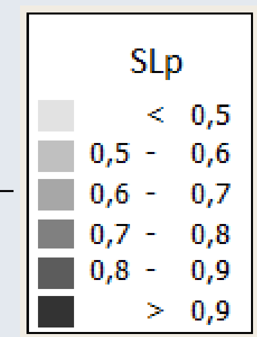


Achieved Service Level

Findings

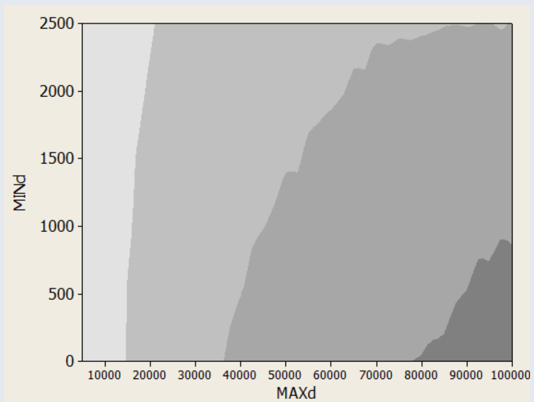
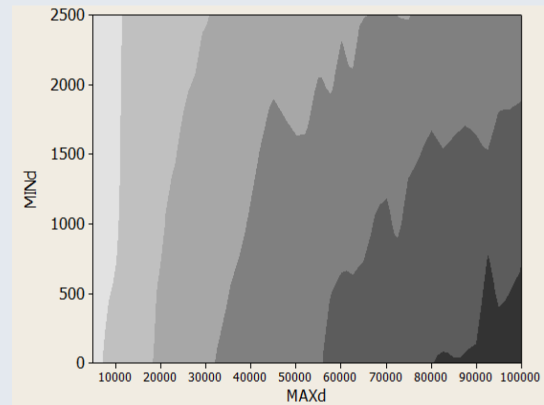
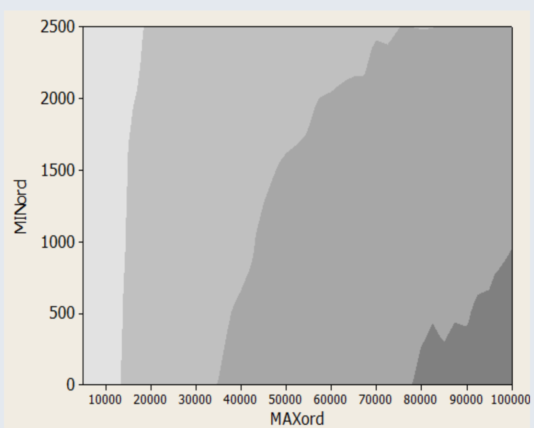
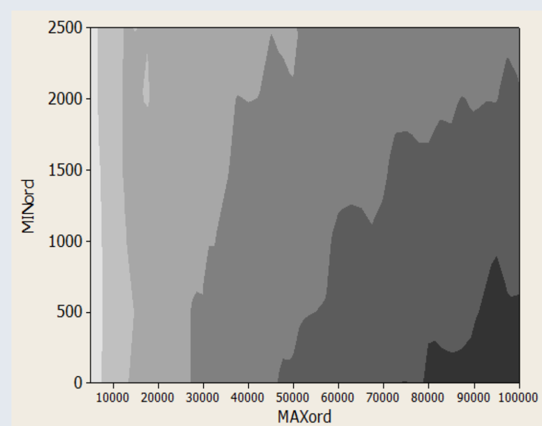
Achieved service level SL_p for different conditions of time sample and imposed monthly σ_i

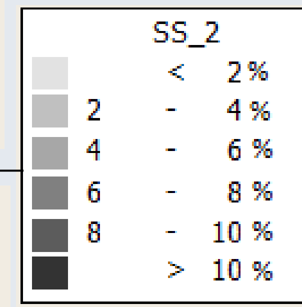
	Sample = Lead time	Sample = a month
Imposed monthly σ_i = 0%		
Imposed monthly σ_i = 20%		



Findings

Calculated safety stocks in terms of D percentage

	Sample = Lead time	Sample = a month
Imposed monthly σ_i = 0%		
Imposed monthly σ_i = 20%		



The used formula for safety stocks calculation is:

$SS = Z_\alpha \sqrt{\sigma_d^2 \, Lt}$

Conclusions

The results shows that:

- **Achieved SL falls when Max_d grows and this effect is grater at small Min_d values**, when the minimum demand admitted in a single day is close to zero.
- **The periodic demand standard deviation σ_i has a small influence on achieved SL.** Comparing a scenario with long term demand stability with a fluctuating one, the results show a slight increment for the achieved SL associated with an increase of the related SS.
- **When Max_d is small the model behavior is very similar to standard EOQ model**.

1. Under relaxed demand assumptions the EOQ model performances are significantly different (lower) from theory.
2. The correlation between service level (SL) and Max_d - Min_d supports the relevance of these two parameters used to characterized demand.
3. Even under demand relaxed assumption, a time sample equal to the lead time is important to assure a good trade-off between SL and SS.
4. In practice the results presented in this paper can be helpful for small-medium manufactures to manage contracts for what it concerns about rewards and penalties related to SL.

Limitations

- Results application is restricted to real cases similar to used parameters set, a sensibility analysis is needed to enlarge results application field;
- No lead time uncertainty is considered in the model;
- The results presented are based on some relaxed assumption about demand, their applicability in real practice have to be proved;
- Different approach ca be used to manage a “non-normal” distributed demand, for example “demand separation” technique. A comparison between different approaches will be useful to quantify expected results in different application fields.

Thanks for your attention !!!

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The EOQ model with normally distributed demand and lead time

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In the EOQ model with:

- normal distributed demand D in any interval of time;
- normal distributed lead time L_t.

The expression for safety stocks is:

$$SS = Z_\alpha \sqrt{\sigma_d^2 \, Lt + \sigma_t^2 \, D^2}$$

where

$$Z_\alpha^{\ *} \propto SL$$

* Z_α is the cumulative probability of the standardized normal random variable